Obviously, the first order principal minor determinant of H is $H_1 < 0$; the second order principal minor determinant of H is $H_2 > 0$; the third order principal minor determinant of H is $H_3 < 0$. That is, the total profit function is convex and there is an optimal solution.

Based on above calculation, the optimal solutions of the channel members in the digital supply chain are listed as follows:

$$
\begin{cases}
i_c^* = \dfrac{-\gamma\left[-6\alpha\beta k_r\sqrt[3]{\theta_1}i_0^2\left(\gamma^2 S-2\beta k_r\right)^2+\gamma\sqrt[3]{12\theta_1^2}-2\sqrt[3]{18}k_m\beta i_0^2\gamma^5\left(2\beta k_r-\gamma^2 S\right)^3\right]}{\sqrt[3]{6}i_0(2\beta k_r-\gamma^2 S)\left[\sqrt[3]{2\theta_1^2}-2\sqrt[3]{3}k_m\beta i_0^2\gamma^4(2\beta k_r-\gamma^2 S)^3\right]} \\[6pt]
p_c^* = \dfrac{\sqrt{\frac{2}{3}}k_m\gamma^5}{\sqrt[3]{\theta_1}} + \dfrac{\alpha k_r}{2\beta k_r-\gamma^2 S} + \dfrac{\gamma\sqrt[3]{\theta_1}}{\sqrt[3]{18}\beta i_0^2(2\beta k_r-\gamma^2 S)^3}, \\[6pt]
h_c^* = \dfrac{2\sqrt[3]{18}k_m\beta i_0^2\gamma^4\left(2\beta k_r-\gamma^2 S\right)^3+\sqrt[3]{\theta_1}\left[3\alpha i_0^2\gamma S\left(\gamma^2 S-2\beta k_r\right)^2-\sqrt[3]{12\theta_1^2}\right]}{3i_0^2(2\beta k_r-\gamma^2 S)^3\sqrt[3]{\theta_1}}
\end{cases}
\tag{1}
$$

Where:

$$
\theta_1 = 9\alpha k_m\beta^2 k_r i_0^4\gamma^3\left(2\beta k_r - \gamma^2 S\right)^5+\sqrt{3}\sqrt{k_m^2\beta^3 i_0^6\gamma^6\left(2\beta k_r-\gamma^2 S\right)^9\left[4k_m\gamma^6+27\alpha^2\beta k_r^2 i_0^2\left(2\beta k_r-\gamma^2 S\right)\right]}
$$

0.1. Stackelberg game model

In the planning period, supplier chooses the appropriate online retailer to cooperate and makes decision on the local advertising effort level with its limited funds, so as to bring potential customers from the stage of considering buying to the stage of desire and action through measures such as reducing prices or improving product quality. In response to the supplier's decision, the online retailer determines the cybersecurity effort level and sets the retail price. If the supplier's local advertising effort level achieves satisfactory decision-making results, correspondingly, more and more customers will register as members on the online retail platform, and online retailer also collects more customer information. Therefore, online retailer must improve cybersecurity effort level to protect customer information from being leaked.