排版

数学写作漫谈

投稿

报告

…

李文威

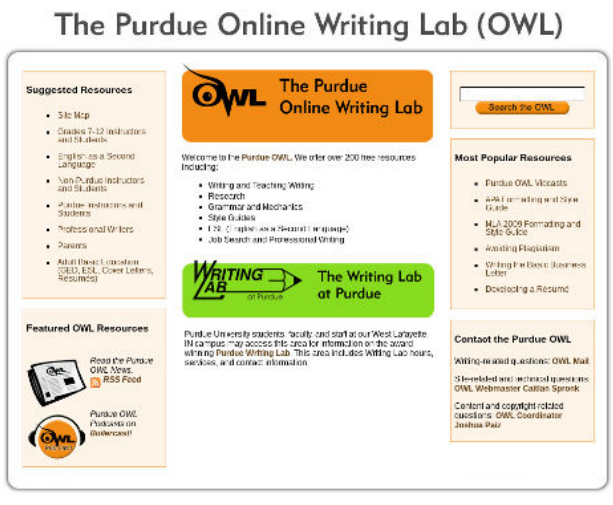
中国科学院数学与系统科学研究院

2014 年 10 月 27 日

释题

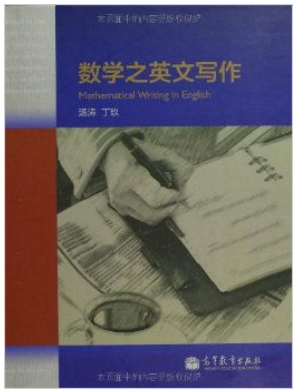
- 学科写作 $\supsetneq$ 科技、工程及数学 (STEM) 写作 $\supsetneq$ 数学写作.
- 科技写作教程/讲座的常见缺点:
  - 无视学科差异;
  - 定位不明 (含选题等);
  - 大处含糊, 小处死板;
  - 商业性重.
- 内部比较: 基础和应用数学的文化各异.
- 对外比较: 与理论物理学和计算机科学相近, 文风活泼.

Harvard Writing Center

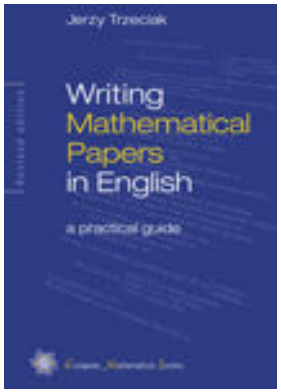


Purdue OWL

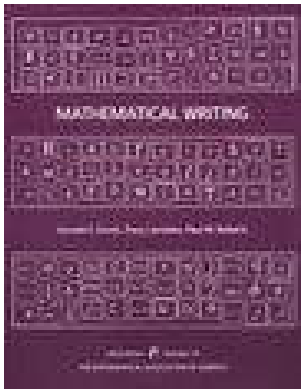
数学写作 $\neq$ 数学英文写作



杨涛, 丁玖



Trzeciak



Knuth

- 常用句, 字句锻炼法 $\rightarrow$ 详阅相关书籍.
- 仅需基本的英文能力 (比较: 日式文章).
- 过度雕饰则有反效果. ("文以气为主")

个人观察: 国人论文文句之混乱, 主要源于观念混乱.

基本方法

操千曲而后晓声，观千剑而后识器.

《文心雕龙 · 知音》

教研究生写作 = 教鱼游泳?

硕博士生未能养成合适的写作风格，原因包括

1. 迷却初心，错认根柢;
2. 风气所限，无意求工;
3. 陈规陋习，师徒相因.

相应地，本讲拟就原理，细节和反例三方面进行剖析.
相关主题: 排版与报告的要点等.

原道

一个中心

写作的目的是交流思想.

两个基本点

体贴 为读者着想, 写作具有“人情味”的文章.

自信 先自问: 能否道人所未曾道? 《文心雕龙 · 神思》: “我才之多少, 将与风云而并驱矣.”

自信体现在作者的

1. 气魄
2. 责任

辨体

数学文章的几种形态: 期刊论文/预印本, 博士论文, 综述, 专著, 教材, 投影片等.

- 不同要素往往共存
  - 稍长的期刊论文既要包含新结果, 又有相当的综述性质.
  - 专著常可兼作教材.
- 读一篇好论文常比读课本能更快习得技术, 缘于其中 (1) 成果 (2) 综述 (3) 教材三要素的调和.
- 网络技术的发展开启新的门径, 如
  - 投影片 (PowerPoint™/Beamer),
  - 报告视频,
  - 博文, 百科条目等.

作为广义的数学写作, 其原理相通.

风格与内容的联系 个人见解:

	长文	短文
偏技术	精密	直接
偏概念	和缓	痛快

经典例子 基础数学领域

- J.-P. Serre, W. Rudin: 精准.
- D. Cox: 和煦亲切.

殊方异俗 如

- 日本: 详尽, 单调
- 美国: 流畅, 张扬
- 英国: 流畅, 节制
- 德国: 精准, 节制
- 法国: 精准, 明快

对文章的形制有疑虑时, 选择**详尽**, **友善**的风格最妥.

- D. Zagier, A One-Sentence Proof That Every Prime $\equiv 1 \pmod 4$ is a Sum of Two Squares (1990) (顾题思义, 主体仅一句话.)

A One-Sentence Proof That Every Prime $p \equiv 1 \pmod 4$ Is a Sum of Two Squares

D. Zagier
Department of Mathematics, University of Maryland, College Park, MD 20742

The involution on the finite set $S = \{(x,y,z) \in \mathbb{N}^3 : x^2 + 4yz = p\}$ defined by

$$(x,y,z) \mapsto \begin{cases} (x+2z,\ z,\ y-x-z) & \text{if } x < y-z \\ (2y-x,\ y,\ x-y+z) & \text{if } y-z < x < 2y \\ (x-2y,\ x-y+z,\ y) & \text{if } x > 2y \end{cases}$$

has exactly one fixed point, so $|S|$ is odd and the involution defined by $(x,y,z) \mapsto (x,z,y)$ also has a fixed point. □

This proof is a simplification of one due to Heath-Brown [1] (inspired, in turn, by a proof given by Liouville). The verifications of the implicitly made assertions—that S is finite and that the map is well-defined and involutory (i.e., equal to its own inverse) and has exactly one fixed point—are immediate and have been left to the reader. Only the last requires that p be a prime of the form $4k+1$, the fixed point then being $(1,1,k)$.

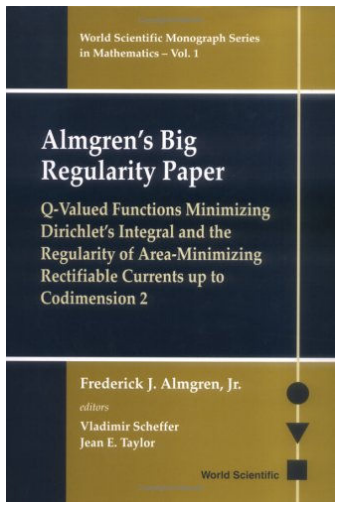
Note that the proof is not constructive: it does not give a method to actually find the representation of p as a sum of two squares. A similar phenomenon occurs with results in topology and analysis that are proved using fixed-point theorems. Indeed, the basic principle we used: "The cardinalities of a finite set and of its fixed-point set under any involution have the same parity," is a combinatorial analogue and special case of the corresponding topological result: "The Euler characteristics of a topological space and of its fixed-point set under any continuous involution have the same parity."

For a discussion of constructive proofs of the two-squares theorem, see the Editor's Corner elsewhere in this issue.

REFERENCE

1. D. R. Heath-Brown, Fermat's two-squares theorem, *Invariant* (1984) 3–5.

- Almgren's Big Regularity Paper (955 页的大 "论文")



常用结构: 载体制约形式, 数学论文的叙述必须是直线的.

- 稍具规模的文章常分为:
  1. 摘要
  2. 目录
  3. 导言
  4. 符号
  5. 主体部分
  6. 参考文献
  7. 附录, 索引等.
- 短文宜直入正题.
- 书籍, 专著等可包含序, 跋或习题.

主体部分留待下节讨论.

摘要:

- 重在达意, 精练为宗.
- 节制公式与符号, 禁引用.

Abstract

Let G be a finite abelian group, and let $m > 0$ with $\exp(G) \mid m$. Let $s_m(G)$ be the generalized Erdős-Ginzburg-Ziv invariant which denotes the smallest positive integer d such that any sequence of elements in G of length d contains a subsequence of length m with sum zero in G. For any integer $r > 0$, let $\mathcal{I}_m^{(r)}$ be the collection of all r-uniform intersecting families of size m. Let $R(\mathcal{I}_m^{(r)}, G)$ be the smallest positive integer d such that any G-coloring of the edges of the complete r-uniform hypergraph $K_d^{(r)}$ yields a zero-sum copy of some intersecting family in $\mathcal{I}_m^{(r)}$. Among other results, we mainly prove that $\Omega(s_m(G)) - 1 \le R(\mathcal{I}_m^{(r)}, G) \le \Omega(s_m(G))$, where $\Omega(s_m(G))$ denotes the least positive integer n such that $\binom{n-1}{r-1} \ge s_m(G)$, and we show that if $r \mid \Omega(s_m(G))-1$ then $R(\mathcal{I}_m^{(r)}, G) = \Omega(s_m(G))$.

反面例子: 您读得下去吗?

目录: LaTeX 可自动生成. 注意深度.

导言: 决定读者观感的关键

1. 标定受众的背景知识, 防低估, 更防高估.
2. 铺垫精简.
   少强调问题的重要性, 不必言必称欧高黎嘉陈.
3. 简述渊源, 精神; 不必强求精确.
4. 文献综述: 重在摘录要旨, 不在按号点名.
5. 最后简述文章的章节结构.

以上亦适用于写书.

D. Knuth, 杰出的计算机科学家, 数学家和段子手.

> *"Another noteworthy characteristic of this manual is that it doesn't always tell the truth."*
>
> *Donald Knuth, Preface to* The TeXbook.

几个反面例子:

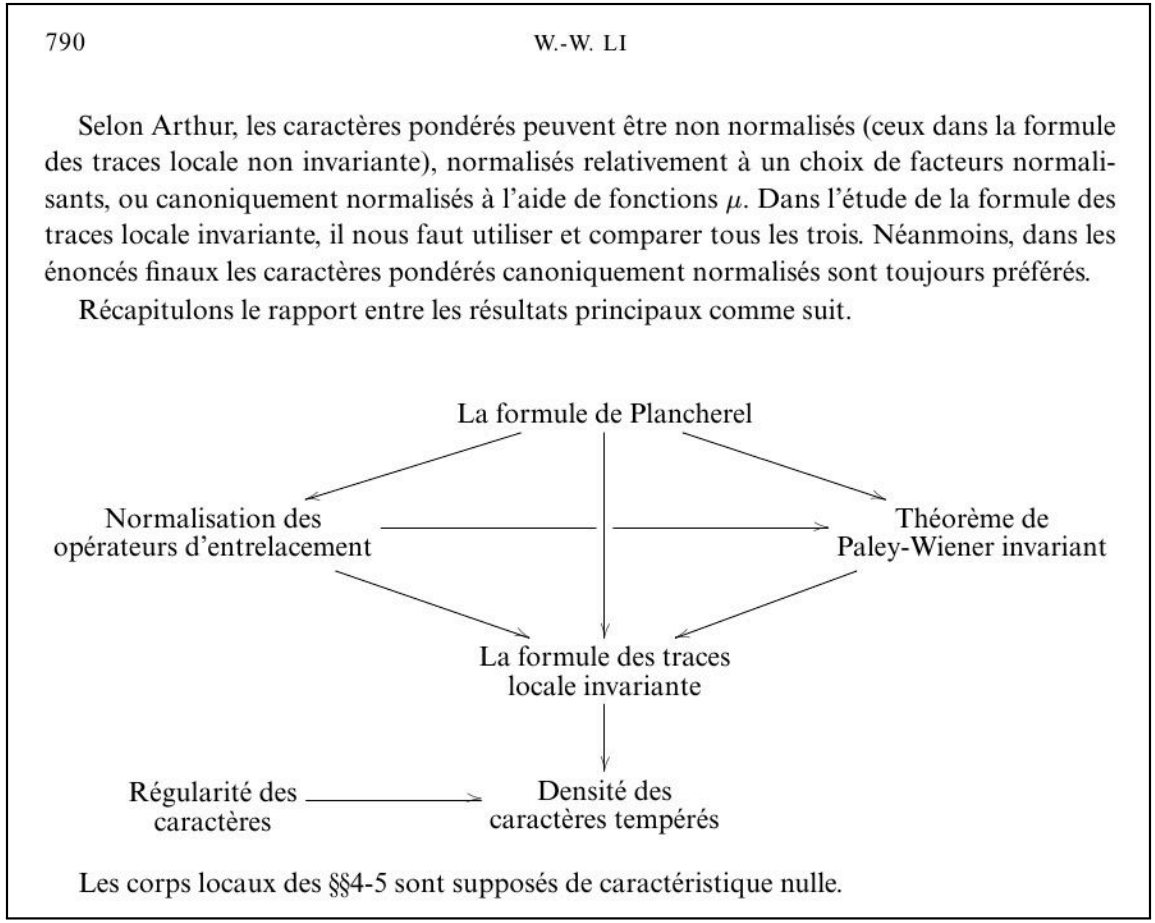
It is well known that the normality of subgroups plays an important part in research in group theory. So it is reasonable to investigate the structure of a group by using normalizers of certain types of subgroups. For example, Bianchi, Gillio Berta Mauri and Hauck in [2] proved that a group is nilpotent if and only if the normalizer of every Sylow subgroup is nilpotent. Ballester-Bolinches and Shemetkov in [1] gave a beautiful criterion for nilpotent groups: a group is nilpotent if and only if the normalizer of every Sylow p-subgroup is p-nilpotent for every prime p.

↑ 劈头便说大道理之一例.

where t is time, $\boldsymbol{x} = x$ in one dimension (1D), i.e., $d = 1$; $\boldsymbol{x} = (x, y)$ in two dimensions (2D), i.e., $d = 2$; and respectively $\boldsymbol{x} = (x, y, z)$ in three dimensions (3D), i.e., $d = 3$, are the Cartesian coordinate; $\Psi(\boldsymbol{x}, t) := (\psi_1(\boldsymbol{x}, t), \psi_2(\boldsymbol{x}, t))^\top$ is the complex-valued macroscopic wave function, $V(\boldsymbol{x})$ is the real-valued external trapping potential, λ is the effective Rabi frequency to realize the internal atomic Josephson junction (JJ) by a Raman transition, δ is the detuning constant for the Raman transition, and $\beta_{jl} = \beta_{lj} = \frac{4\pi N a_{jl}}{a_0}$ $(j, l = 1, 2)$ are interaction constants with N being the total number of particles in the two-component BEC, a_0 being the dimensionless spatial unit, and $a_{jl} = a_{lj}$ $(j, l = 1, 2)$ being the s-wave scattering lengths between the j-th and l-th component (positive for repulsive interaction and negative for attractive interaction). From the physical point of view, it is necessary to ensure that the wave function is properly normalized-specifically, we require

↑ 导言里符号和定义过多之一例.

可图示各章节或概念间的关系. 此法常见于书籍, 也适用较长的或较复杂的论文.



790 W.-W. LI

Selon Arthur, les caractères pondérés peuvent être non normalisés (ceux dans la formule des traces locale non invariante), normalisés relativement à un choix de facteurs normalisants, ou canoniquement normalisés à l'aide de fonctions μ. Dans l'étude de la formule des traces locale invariante, il nous faut utiliser et comparer tous les trois. Néanmoins, dans les énoncés finaux les caractères pondérés canoniquement normalisés sont toujours préférés.

Récapitulons le rapport entre les résultats principaux comme suit.

Les corps locaux des §§4-5 sont supposés de caractéristique nulle.

例子: L., *La formule des traces pour les revêtements de groupes réductifs connexes II.*, Ann. Scient. ENS, t. 45(2012).

符号列表: 集中列举文章用到的一般符号和假设, 作者读者两利. 例如: 假设域的特征是零等等...

正面例子: T. Finis, E. Lapid and W. Müller, *On the spectral side of Arthur's trace formula — absolute convergence* (Ann. Math. **174**, 2011)

> 2.1. **Notation.** Let G be a reductive group defined over a number field F. All algebraic subgroups of G considered in the following will be tacitly assumed to be defined over F. We will mostly use, with some minor modifications, the notation and conventions of [Art82a, Art82b]. In particular:
>
> - $\mathbb{A}$ is the ring of adeles of F, $\mathbb{A}_f$ the ring of finite adeles and $F_\infty = \mathbb{R} \otimes_{\mathbb{Q}} F$.
> - $\mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ is the universal enveloping algebra of the complexified Lie algebra of $G(F_\infty)$.
> - $\mathfrak{z}$ is the center of $\mathcal{U}(\mathfrak{g}_{\mathbb{C}})$.
> - T_0 is a fixed maximal F-split torus.
> - M_0 is the centralizer of T_0, which is a minimal Levi subgroup defined over F.
>
> [1]Note the following typo on p. 15564 of [ibid.]: the *sum* over $s \in W(M)$ should be replaced by the *average* over $s \in W(M)$.

未完待续...

- A_0 is the identity component of $T_0(\mathbb{R})$, which is viewed as a subgroup of $T_0(\mathbb{A})$ via the diagonal embedding of $\mathbb{R}$ into F_∞.
- $\mathcal{L}$ is the set of Levi subgroups containing M_0, i.e. the (finite) set of centralizers of subtori of T_0.
- $W_0 = N_{G(F)}(T_0)/M_0$ is the Weyl group of (G, T_0), where $N_{G(F)}(H)$ is the normalizer of H in $G(F)$.
- For any $s \in W_0$ we choose a representative $w_s \in G(F)$.
- W_0 acts on $\mathcal{L}$ by $sM = w_s M w_s^{-1}$.

For $M \in \mathcal{L}$ we use the following additional notation.

- T_M is the split part of the identity component of the center of M.
- $W(M) = N_{G(F)}(M)/M$, which can be identified with a subgroup of W_0.
- $A_M = A_0 \cap T_M(\mathbb{R})$.
- $\mathfrak{a}_M^*$ is the $\mathbb{R}$-vector space spanned by the lattice $X^*(M)$ of F-rational characters of M; $\mathfrak{a}_{M,\mathbb{C}}^* = \mathfrak{a}_M^* \otimes_{\mathbb{R}} \mathbb{C}$.
- $\mathfrak{a}_M$ is the dual space of $\mathfrak{a}_M^*$, which is spanned by the co-characters of T_M.
- $H_M : M(\mathbb{A}) \to \mathfrak{a}_M$ is the homomorphism given by $e^{\langle \chi, H_M(m)\rangle} = |\chi(m)|_{\mathbb{A}} = \prod_v |\chi(m_v)|_v$ for any $\chi \in X^*(M)$.
- $M(\mathbb{A})^1 \subset M(\mathbb{A})$ is the kernel of H_M.
- $\mathcal{L}(M)$ is the set of Levi subgroups containing M.
- $\mathcal{P}(M)$ is the set of parabolic subgroups of G with Levi part M.
- $\mathcal{F}(M) = \mathcal{F}^G(M) = \coprod_{L \in \mathcal{L}(M)} \mathcal{P}(L)$ is the (finite) set of parabolic subgroups of G containing M.
- $W(M)$ acts on $\mathcal{P}(M)$ and $\mathcal{F}(M)$ by $sP = w_s P w_s^{-1}$.
- Σ_M is the set of reduced roots of T_M on the Lie algebra of G.
- For any $\alpha \in \Sigma_M$ we denote by $\alpha^\vee \in \mathfrak{a}_M$ the corresponding co-root.
- $L^2_{\mathrm{disc}}(A_M M(F)\backslash M(\mathbb{A}))$ is the discrete part of $L^2(A_M M(F)\backslash M(\mathbb{A}))$, i.e. the closure of the sum of all irreducible subrepresentations of the regular representation of $M(\mathbb{A})$.
- $\Pi_{\mathrm{disc}}(M(\mathbb{A}))$ denotes the countable set of equivalence classes of irreducible unitary representations of $M(\mathbb{A})$ which occur in the decomposition of $L^2_{\mathrm{disc}}(A_M M(F)\backslash M(\mathbb{A}))$ into irreducibles.

For any $L \in \mathcal{L}(M)$ we identify $\mathfrak{a}_L^*$ with a subspace of $\mathfrak{a}_M^*$. We denote by $\mathfrak{a}_M^L$ the annihilator of $\mathfrak{a}_L^*$ in $\mathfrak{a}_M$. For any integer $i \geq 0$ let

$$\mathcal{L}_i(M) = \{L \in \mathcal{L}(M) : \dim \mathfrak{a}_M^L = i\}$$

and

$$\mathcal{F}_i(M) = \bigcup_{L \in \mathcal{L}_i(M)} \mathcal{P}(L),$$

so that $\mathcal{F}(M) = \coprod_{i=0}^d \mathcal{F}_i(M)$ where d is the co-rank of M. We endow $\mathfrak{a}_{M_0}$ with the structure of a Euclidean space by choosing a W_0-invariant inner product. This choice fixes Haar measures on the spaces $\mathfrak{a}_M^L$ and their duals $(\mathfrak{a}_M^L)^*$. We follow Arthur in the corresponding normalization of Haar measures on the groups $M(\mathbb{A})$ ([Art78, §1]).

For any $P \in \mathcal{P}(M)$ we use the following notation.

- $\mathfrak{a}_P = \mathfrak{a}_M$.
- N_P is the unipotent radical of P and M_P is the unique $L \in \mathcal{L}(M)$ (in fact the unique $L \in \mathcal{L}(M_0)$) such that $P \in \mathcal{P}(L)$.
- $\Sigma_P \subset \mathfrak{a}_P^*$ is the set of reduced roots of T_M on the Lie algebra of N_P.
- Δ_P is the subset of simple roots of P, which is a basis for $(\mathfrak{a}_P^G)^*$.
- $\mathfrak{a}_{P,+}^*$ is the closure of the Weyl chamber of P, i.e.

$$\mathfrak{a}_{P,+}^* = \{\lambda \in \mathfrak{a}_M^* : \langle \lambda, \alpha^\vee\rangle \geq 0 \text{ for all } \alpha \in \Sigma_P\} = \{\lambda \in \mathfrak{a}_M^* : \langle \lambda, \alpha^\vee\rangle \geq 0 \text{ for all } \alpha \in \Delta_P\}.$$

- δ_P is the modulus function of $P(\mathbb{A})$.
- v_{Δ_P} is the co-volume of the lattice spanned by Δ_P in $(\mathfrak{a}_P^G)^*$ and

$$\theta_P(\lambda) = v_{\Delta_P}^{-1} \prod_{\alpha \in \Delta_P} \langle \lambda, \alpha^\vee\rangle, \quad \lambda \in \mathfrak{a}_{M,\mathbb{C}}^*.$$

- $P^\circ \in \mathcal{P}(M)$ is the parabolic subgroup opposite to P (with respect to M), i.e. $\Sigma_{P^\circ} = -\Sigma_P$ and $\Delta_{P^\circ} = -\Delta_P$.
- $\bar{A}_2(P)$ is the Hilbert space completion of

$$\{\phi \in C^\infty(M(F)N_P(\mathbb{A})\backslash G(\mathbb{A})) : \delta_P^{-\frac{1}{2}}\phi(\cdot x) \in L^2_{\mathrm{disc}}(A_M M(F)\backslash M(\mathbb{A})) \ \forall x \in G(\mathbb{A})\}$$

with respect to the inner product

$$(\phi_1, \phi_2) = \int_{A_M M(F)N_P(\mathbb{A})\backslash G(\mathbb{A})} \phi_1(g)\overline{\phi_2(g)}\, dg.$$

Let $\alpha \in \Sigma_M$. We say that two parabolic subgroups $P, Q \in \mathcal{P}(M)$ are *adjacent* along α, and write $P|^\alpha Q$, if $\Sigma_P \cap \Sigma_{Q^\circ} = \{\alpha\}$. Alternatively, P and Q are adjacent if the closure $\overline{PQ}$ of PQ belongs to $\mathcal{F}_1(M)$. Any $R \in \mathcal{F}_1(M)$ is of the form $\overline{PQ}$ for a unique unordered pair $\{P, Q\}$ of parabolic subgroups in $\mathcal{P}(M)$, namely P and Q are the maximal parabolic subgroups of R, and $P|^\alpha Q$ with $\alpha^\vee \in \Sigma_P^\vee \cap \mathfrak{a}_M^R$. Switching the order of P and Q changes α to $-\alpha$.

2.2. **Intertwining operators.** Fix a maximal compact subgroup $K = K_\infty K_f$ of $G(\mathbb{A}) = G(F_\infty)G(\mathbb{A}_f)$ which is admissible with respect to M_0. For any $P \in \mathcal{P}(M)$ let

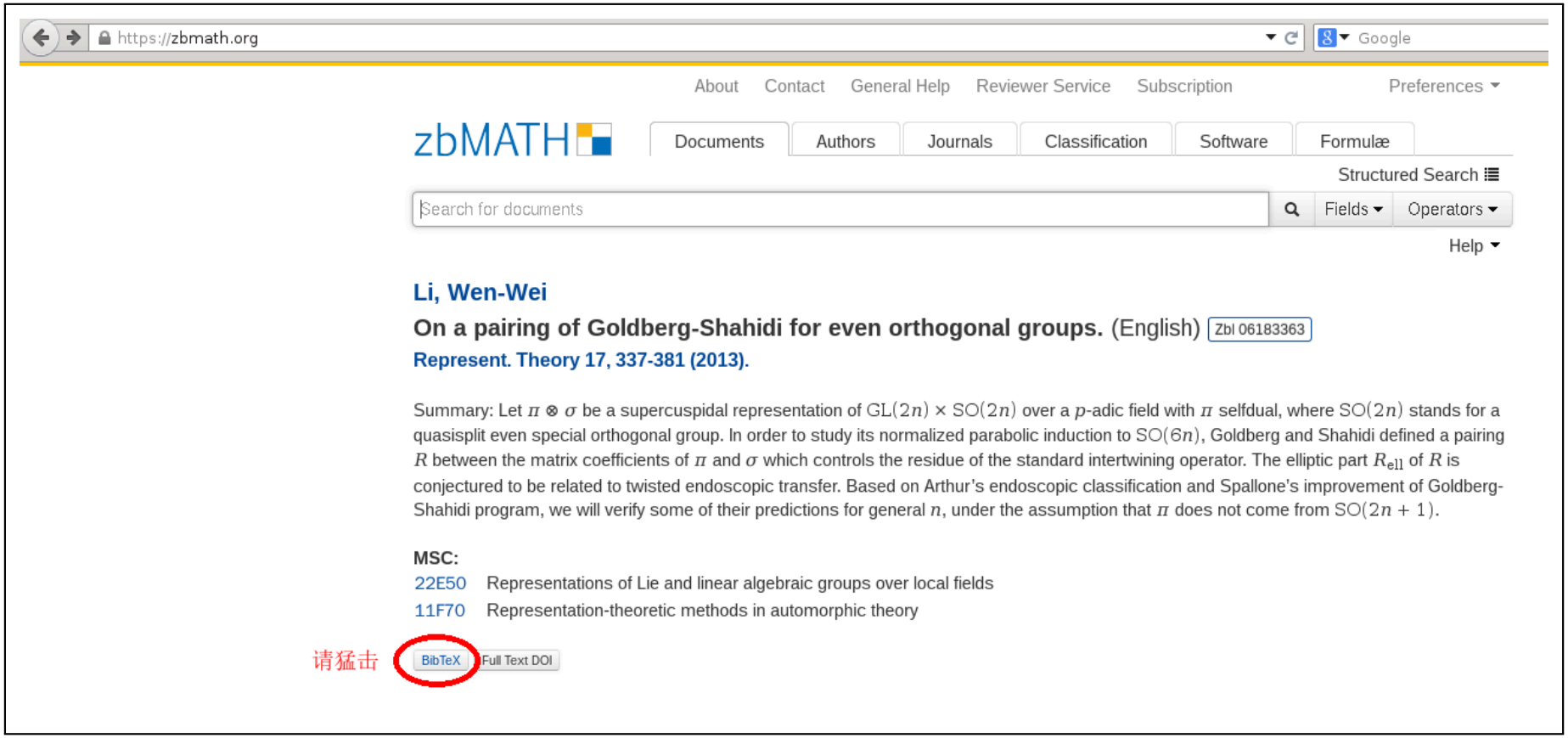
- $H_P : G(\mathbb{A}) \to \mathfrak{a}_P$ be the extension of H_M to a left $N_P(\mathbb{A})$- and right K-invariant map,
- $\mathcal{A}^2(P)$ the dense subspace of $\bar{\mathcal{A}}^2(P)$ consisting of its K- and $\mathfrak{z}$-finite vectors, i.e. the space of automorphic forms ϕ on $N_P(\mathbb{A})M(F)\backslash G(\mathbb{A})$ such that $\delta_P^{-\frac{1}{2}}\phi(\cdot k)$ is a square-integrable automorphic form on $A_M M(F)\backslash M(\mathbb{A})$ for all $k \in K$,
- $\rho(P, \lambda)$, $\lambda \in \mathfrak{a}_{M,\mathbb{C}}^*$, the induced representation of $G(\mathbb{A})$ on $\bar{\mathcal{A}}^2(P)$ given by

$$(\rho(P, \lambda, y)\phi)(x) = \phi(xy)e^{\langle \lambda, H_P(xy) - H_P(x)\rangle}.$$

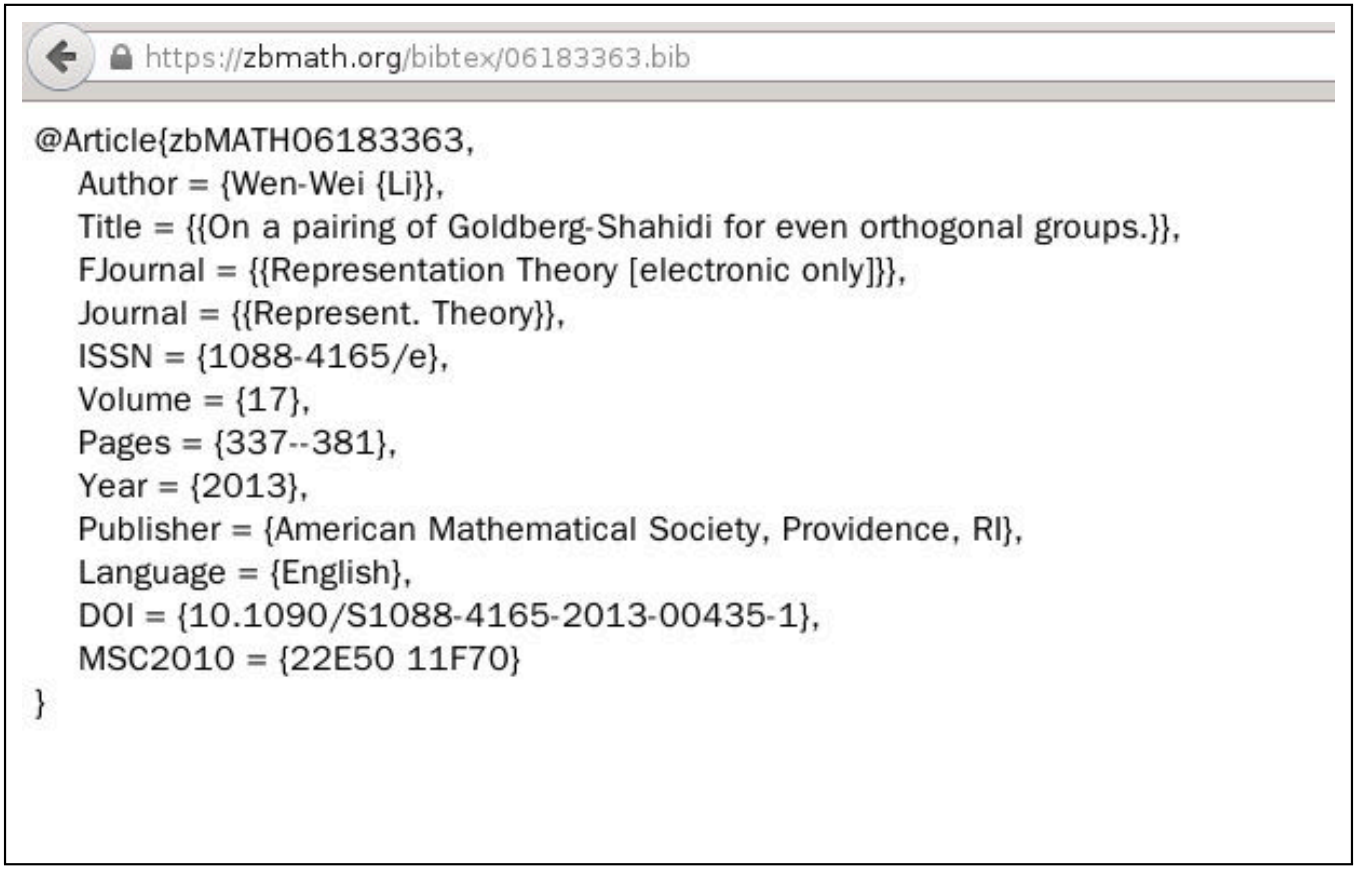
It is isomorphic to $\mathrm{Ind}_{P(\mathbb{A})}^{G(\mathbb{A})}\left(L^2_{\mathrm{disc}}(A_M M(F)\backslash M(\mathbb{A})) \otimes e^{\langle \lambda, H_M(\cdot)\rangle}\right)$.

符号们都安静地躺着.

参考文献: 无论多寡, 都建议用 BibTeX 自动生成.
取得规范, 详细的 BibTeX 资料的第一种方法: 先在 zbMATH 找到条目

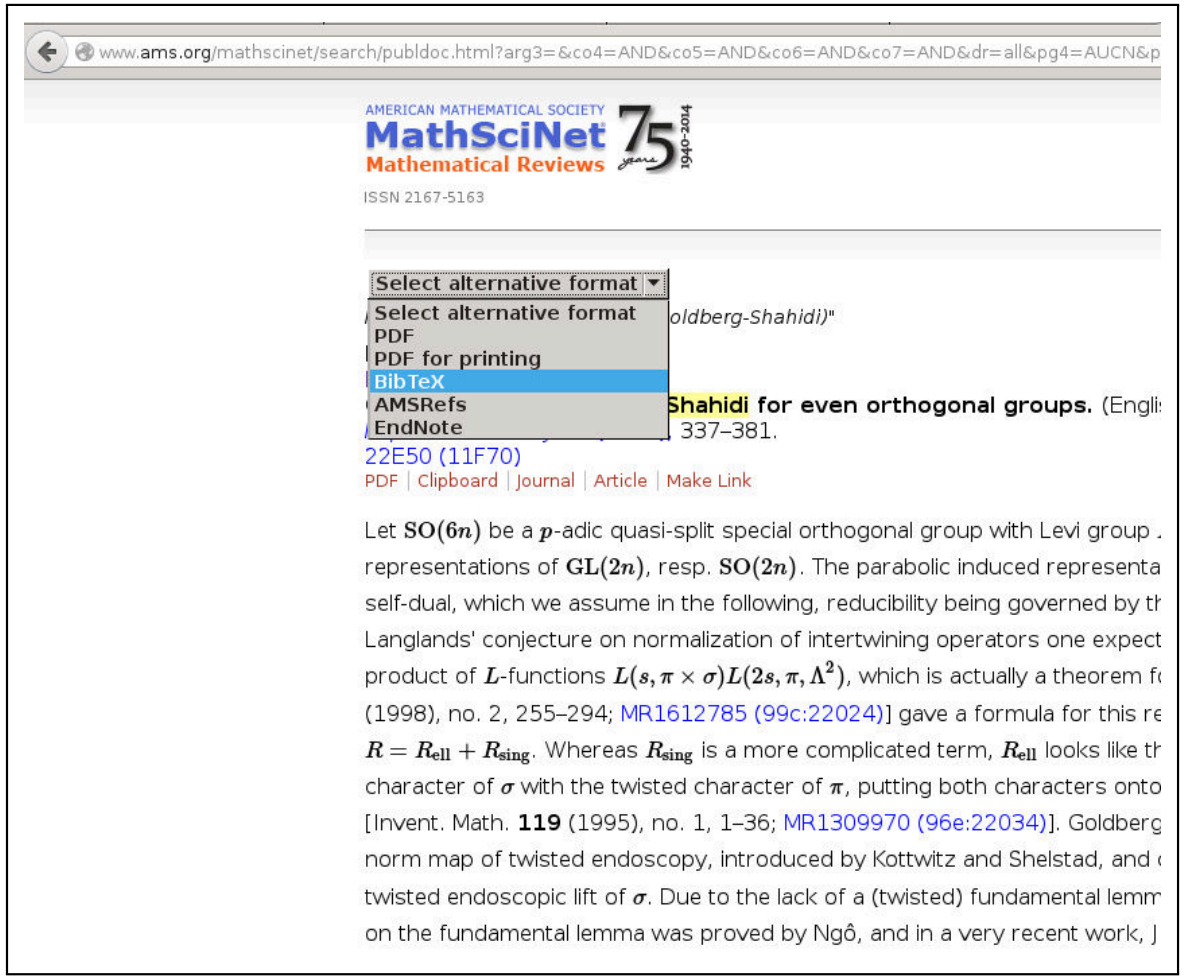


结果:

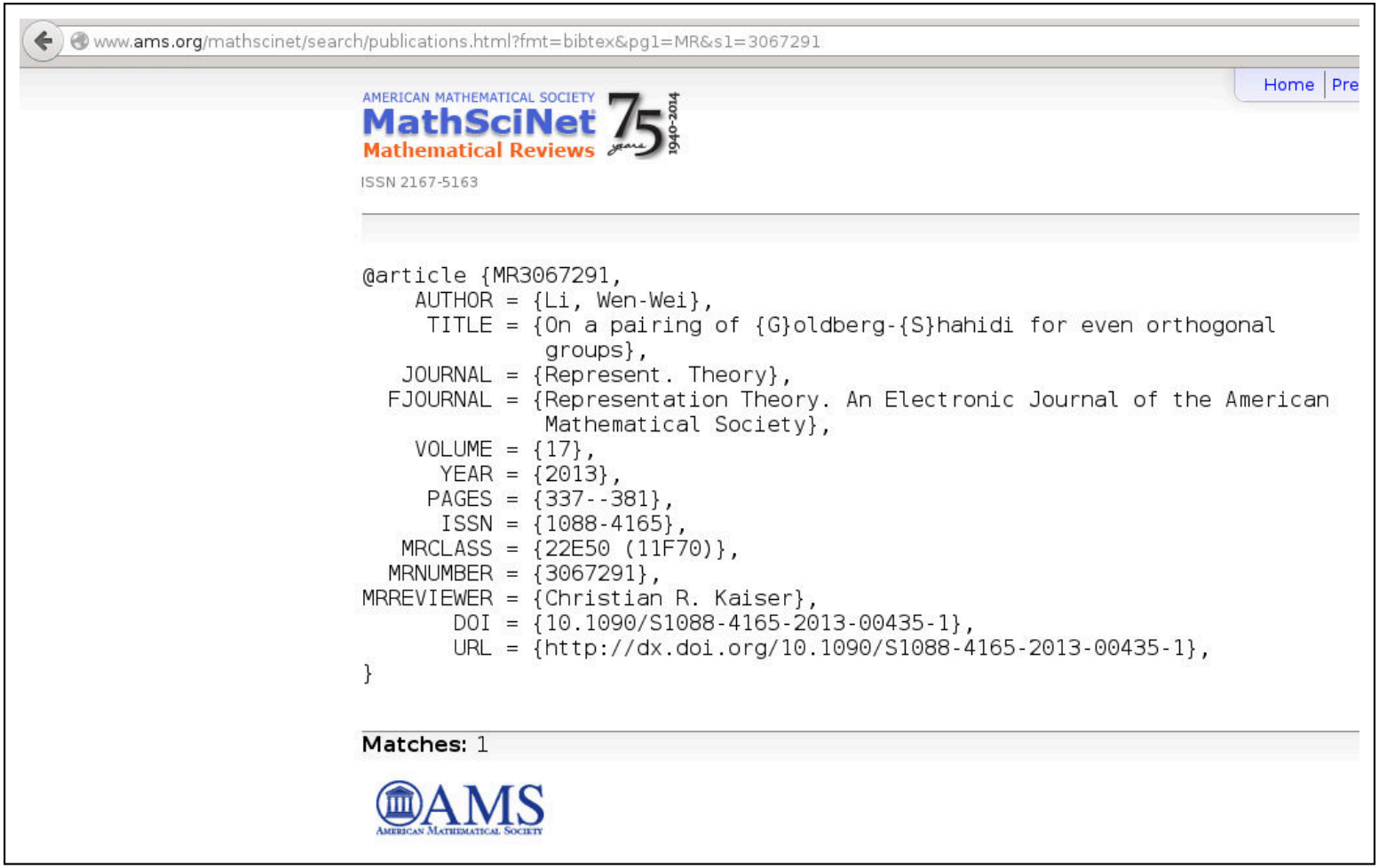


复制到自己的 BibTeX 文献库, 并将代码 zbMATH06183363 修改成合适的名称, 即可使用.

准此要领, 从 MathSciNet 亦可获取:



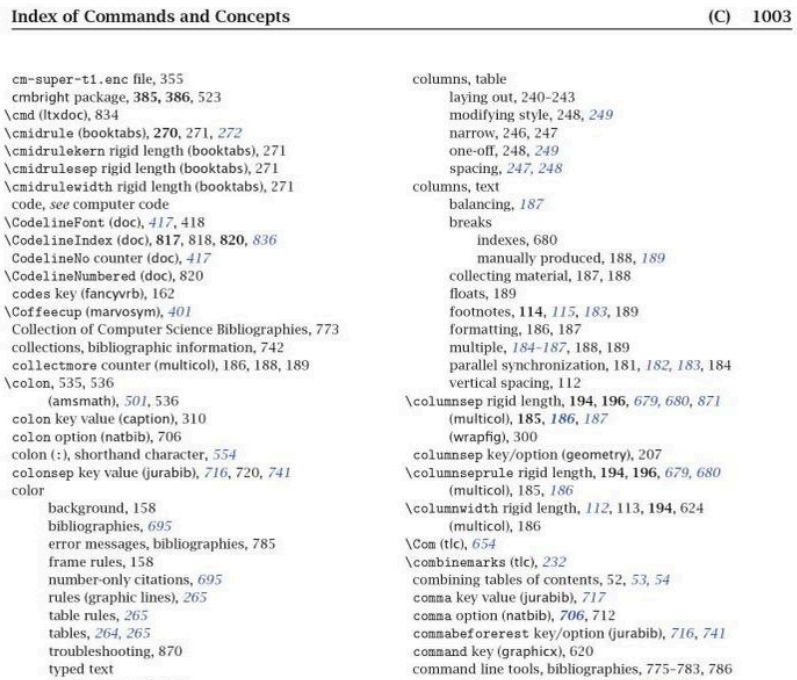
结果:



同样再复制粘贴. 正常网速下耗时不过半分钟. 其它自动获得文献信息的方法: Google Scholar 等等...

索引: 50 页以上的论文都 “应当” 有索引, 然而愿意制作者尚少.

- 在 LaTeX 下可使用 *MakeIndex* 或 `xindy` 等程序编纂索引. 易学易用.
- 个人经验: 为 80 页的论文添加索引, 仅费不到半小时.



Index of Commands and Concepts (C) 1003

The LaTeXCompanion (1090 页) 索引一瞥.

析用

如何铺陈证明和论述?

- 注重动机, 长篇大论前尽量提示背景和想法.
- 控制节奏
  - 过渡处宜适当插入词句, 增加流畅性.
  - 数学写作的根本价值在于易读易懂, 而非简省字句; 后者是为前者服务的.
  - 避免语气急促的并列句.
  - 用词变化, 避免千篇一律 (如 thus... so... thus... so...).
- 引文: 给出页码或编号, 除非所引文章甚短, 或只为证明该结果.

It is clear from the definitions that the summand is indeed invariant under left translation of u by S, that $J_A\phi$ belongs to $\mathcal{H}_{S'}(\sigma')$ as a function of x, and that as a linear transformation from $\mathcal{H}_S(\sigma)$ to $\mathcal{H}_{S'}(\sigma)$, J_A satisfies

$$J_A\mathcal{I}_S(\sigma, y) = \mathcal{I}_{S'}(\sigma', y)J_A, \qquad y \in H.$$

Mackey's result is as follows:

PROPOSITION 2.1 (Mackey[**M1**]). *The correspondence*

$$A \longrightarrow J_A$$

is an isomorphism from the vector space of linear transformations that satisfy (2.1) to the vector space of intertwining operators from $\mathcal{I}_S(\sigma)$ to $\mathcal{I}_{S'}(\sigma')$.

The proof of the proposition is elementary. Despite its simplicity, however, the correspondence $A \to J_A$ can be seen as a model for later work that is now at the heart of representation theory. This includes Harish-Chandra's theory of the Eisenstein integral, an important part of local harmonic analysis, Langlands' theory of Eisenstein series, which led to the general theory of automorphic forms, and the work of a number of people on the intertwining operators among the induced representations $\{\mathcal{I}_P(\pi)\}$. It is the last of these three topics that will be our concern here.

The domain of Mackey's correspondence is easy to characterize. As a vector

思考题

以上引文中, 哪些句子可以略去? 精简后的版本是否更顺畅, 更易读?

J. Arthur, *Induced representations, intertwining operators and transfer*, Contemp. Math. 449.

- 让读者厌烦的不只是局促结巴的 (中式) 并列句, 还有拥挤的公式. 例如:

Let $\{P'_1, \cdots, P'_{r'}\}$ (resp. $\{P_1, \cdots, P_r\}$, $\{P''_1, \cdots, P''_{r''}\}$) be a complete set of indecomposable projective A'-modules (resp. A-modules, A''-modules). Then their tops $\{S'_1, \cdots, S'_{r'}\}$ (resp. $\{S_1, \cdots, S_r\}$, $\{S''_1, \cdots, S''_{r''}\}$) is a complete set of simple A'-modules (resp. A-modules, A''-modules). By [8, Theorem 1.1] or [1, Proposition 6.5], we have $r' + r'' = r$.

Consider the triangles $j_!j^!P_u \to P_u \to i_*i^*P_u \to$ for all $1 \leq u \leq r$. Since $P_u \in K^b(\mathrm{proj}A)$, we have $j^!P_u \in K^b(\mathrm{proj}A'') = \mathrm{tria}\{P''_1, \cdots, P''_{r''}\} \subseteq \mathcal{D}A''$ and $i^*P_u \in K^b(\mathrm{proj}A') = \mathrm{tria}\{P'_1, \cdots, P'_{r'}\} \subseteq \mathcal{D}A'$. Here, for a class $\mathcal{X}$ of objects in a triangulated category $\mathcal{T}$, $\mathrm{tria}\mathcal{X}$ denotes the smallest full triangulated subcategory of $\mathcal{T}$ containing $\mathcal{X}$. Furthermore, we have $j_!j^!P_u \in \mathrm{tria}\{j_!P''_1, \cdots, j_!P''_{r''}\} \subseteq \mathcal{D}A$ and $i_*i^*P_u \in \mathrm{tria}\{i_*P'_1, \cdots, i_*P'_{r'}\} \subseteq \mathcal{D}A$. Hence $P_u \in \mathrm{tria}\{i_*P'_1, \cdots, i_*P'_{r'}, j_!P''_1, \cdots, j_!P''_{r''}\} \subseteq \mathcal{D}A$, and $K^b(\mathrm{proj}A) = \mathrm{tria}\{P_1, \cdots, P_r\} = \mathrm{tria}\{i_*P'_1, \cdots, i_*P'_{r'}, j_!P''_1, \cdots, j_!P''_{r''}\} \subseteq \mathcal{D}A$. Therefore, $\{[i_*P'_1], \cdots, [i_*P'_{r'}], [j_!P''_1], \cdots, [j_!P''_{r''}]\}$ is a $\mathbb{Z}$-basis of $K_0(K^b(\mathrm{proj}A))$.

Consider the triangles $i_*i^!S_u \to S_u \to j_*j^!S_u \to$ for all $1 \leq u \leq r$. Since $S_u \in \mathcal{D}^b(\mathrm{mod}A)$, we have $i^!S_u \in \mathcal{D}^b(\mathrm{mod}A') = \mathrm{tria}\{S'_1, \cdots, S'_{r'}\} \subseteq \mathcal{D}A'$ and $j^!S_u \in \mathcal{D}^b(\mathrm{mod}A'') = \mathrm{tria}\{S''_1, \cdots, S''_{r''}\} \subseteq \mathcal{D}A''$. Furthermore, we have

- 局促的公式 + 局促的语句 $\Rightarrow$ 局促的节奏 $\Rightarrow$ 难受的阅读体验.
- 别吝于使用显示公式!

交叉参照的妙用

- 主要矛盾
  - 读者的精力和注意力有限, 且时间零碎,
  - 而文章环环相扣, 经纬万端.
- 叙事技巧
  - **伏笔** — 提示所证结果在全文中的地位 (用处), 和其它结果的联系等.
  - **闪回** — 前情提要, 或回顾稍早的理论框架和已证结果等.
    伏笔和闪回特别适用于各章节的开头.
  - **直入正题** (*in medias res*) — 可用于导言.

对比: 小说, 电影, 评书的叙事手法.

> 启行之辞, 逆萌中篇之意; 绝笔之言, 追媵前句之旨; 故能外文绮交, 内义脉注, 跗萼相衔, 首尾一体.
>
> 《文心雕龙 · 章句》

向程序员学习

- 模块化,
- 封装,
- 权变 (!): 如在编程中使用 `goto`, 或在论文中引用证明.

"The practitioner of literate programming can be regarded as an essayist, whose main concern is with exposition and excellence of style. Such an author, with thesaurus in hand, chooses the names of variables carefully and explains what each variable means. He or she strives for a program that is comprehensible because its concepts have been introduced in an order that is best for human understanding, using a mixture of formal and informal methods that reinforce each other."

D. Knuth, Literate Programming, *The Computer Journal 27(2):97–111, 1984.*

"I look forward to the day when a Pulitzer Prize will be given for the best computer program of the year."

D. Knuth, Mathematical Writings, *§10.*

其它建议

1. 符号要精挑细选 (Leibniz, ...). 经典渣例子 (Barry Mazur):

$$\Big[\Xi \,\Big|\, \Xi\Big] \qquad (\Xi \in \mathbb{C}^{\times}).$$

2. 读诵文句以检查韵律.
3. 避开抄袭**嫌疑**:

> arXiv admin note: substantial text overlap with arXiv:[redacted] by other authors

4. 多读 Bulletin of the AMS 或 Notices of the AMS, 学习**讲人话**.

BULLETIN (NEW SERIES) of the AMERICAN MATHEMATICAL SOCIETY

The typesetting of this work will be sensitive to small differences in notation. Although no difficult simultaneous comparison between similar symbols will be required, the differences should not be overlooked when looking for references. More concretely, we have used all the following fonts: A (normal), A (Roman), $\mathbf{A}$ (boldface), $\mathbb{A}$ (blackboard boldface), A (sans serif), $\mathtt{A}$ (typewriter), $\mathcal{A}$ (calligraphic), $\mathfrak{A}$ (Fraktur), and $\mathscr{A}$ (Ralph Smith's formal script). The tiny difference between A (normal) and A (italic) in width, which does exist, seems to be extremely difficult to see. So we shall never use both of them. We distinguish between A and $\underline{A}$, where the latter almost always means the relative version of A (as a sheaf or functor, etc.). We distinguish between Greek letters in each of the pairs ϵ and ε, ρ and ϱ, σ and ς, ϕ and φ, and π and ϖ. The musical symbols $^\flat$ (flat), $^\natural$ (natural), and $^\sharp$ (sharp) will be used following Grothendieck (cf., for example, [61, IX]) and some other authors. The difference in each of the pairs $^\flat$ and ^{b}, and $^\sharp$ and $^\#$, should not lead to any confusion. The notation $^\heartsuit$ and $^\diamondsuit$ are used, respectively, for Mumford families and good formal models, where the convention for the former follows from [46]. We distinguish between the two star signs * and *. The two dagger forms † and ‡ are used as superscripts. The differences between v, ν, υ, and the dual sign $^\vee$ should not be confusing because they are never used for similar purposes. The same is true for i, ι, $\imath$, and $\jmath$. Since we will never need calculus in this work, the symbols ∂, $\int$, and $\oint$ are used as variants of d or S.

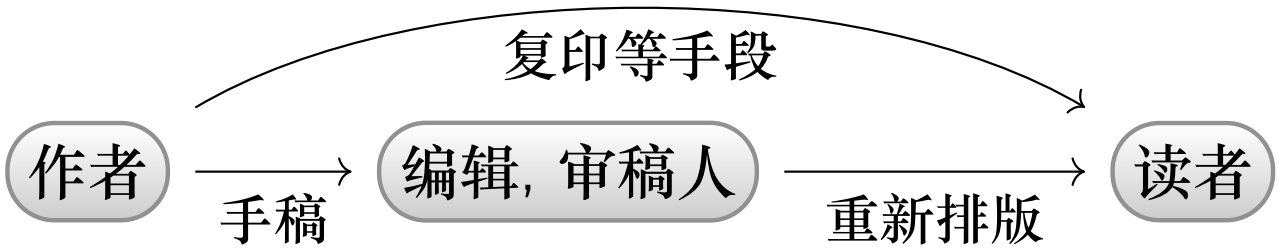
↑ 符号狂之一例 (K.-W. Lan, *Arithmetic Compactifications of PEL-Type Shimura Varieties*, 2013).

论形

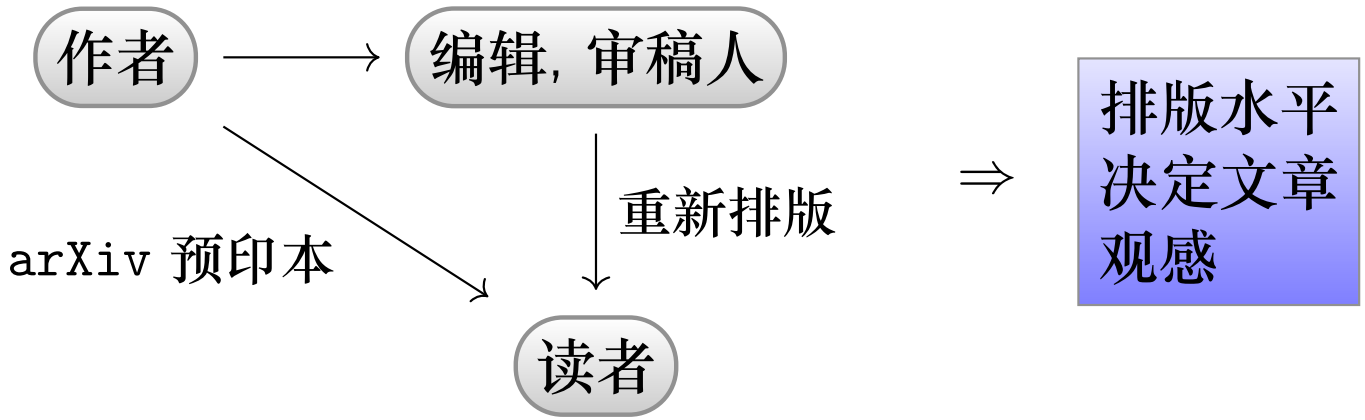
Typography is what language looks like.

— *Ellen Lupton*, Thinking with Type, *2nd ed.*

1. 传统模式:

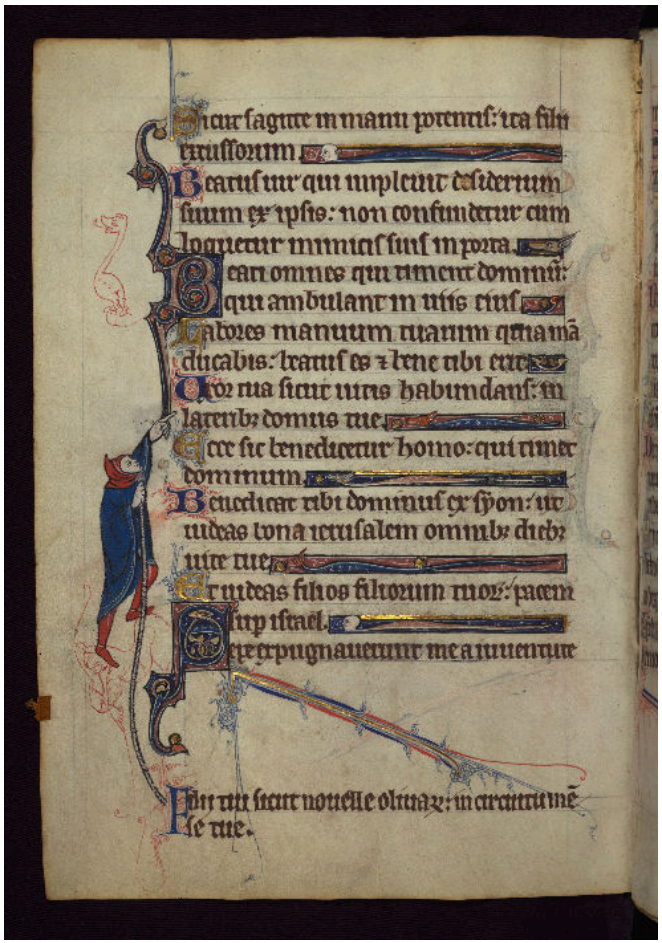


2. 网络时代:



从中世纪手抄本到铅字排版

来源: The Digital Walters | Whittaker and Watson | Wikimedia Commons



Let C be a contour in the z-plane, and let $f(z)$ be a function a
Let $\phi(z)$ be another function which is analytic inside and on C e
of poles; let the zeros of $\phi(z)$ in the interior* of C be $a_1, a_2, \ldots,$
multiplicity be $r_1, r_2, \ldots$; and let its poles in the interior of C b
degrees of multiplicity be $s_1, s_2, \ldots$.

Then, by the fundamental theorem of residues, $\frac{1}{2\pi i}\int_C f(z)\frac{\phi'(z}{\phi(z}$
of the residues of $\frac{f(z)\,\phi'(z)}{\phi(z)}$ at its poles inside C.

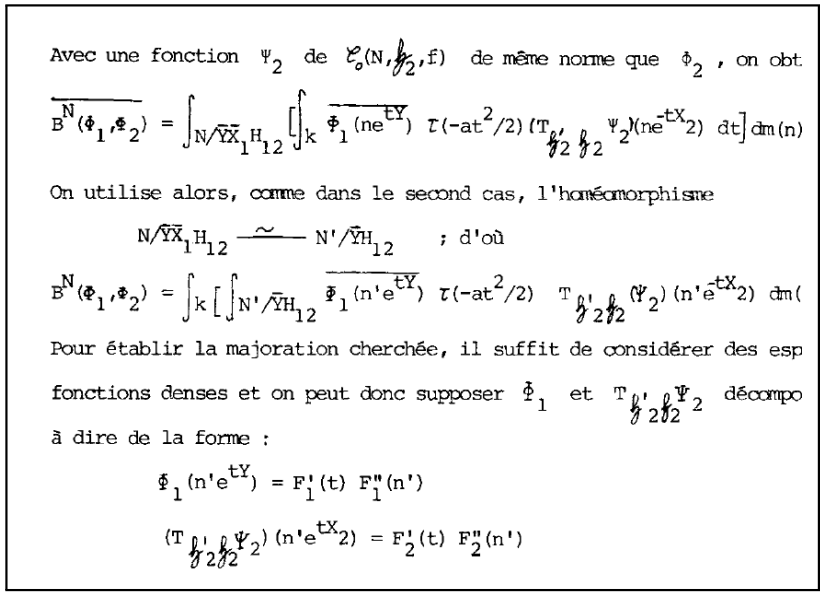
Now $\frac{f(z)\,\phi'(z)}{\phi(z)}$ can have singularities only at the poles and z
of the zeros, say a_1, we have

$$\phi(z) = A(z-a_1)^{r_1} + B(z-a_1)^{r_1+1} + \ldots.$$

Therefore
$$\phi'(z) = Ar_1(z-a_1)^{r_1-1} + B(r_1+1)(z-a_1)^{r_1} + .$$

and
$$f(z) = f(a_1) + (z-a_1)f'(a_1) + \ldots.$$

打字机 (1981) 对决 LaTeX(2014)



Avec une fonction Ψ_2 de $\mathcal{C}_o(N,\mathfrak{f}_2,f)$ de même norme que Φ_2 , on obt

$$\overline{B^N(\Phi_1,\Phi_2)} = \int_{N/\bar{Y}\bar{X}_1H_{12}} \Big[\int_k \overline{\Phi_1(ne^{tY})}\ \tau(-at^2/2)\,(T_{\mathfrak{f}'_2\mathfrak{f}_2}\Psi_2)(ne^{-tX}2)\ dt\Big]\,dm(n)$$

On utilise alors, comme dans le second cas, l'homéomorphisme

$$N/\bar{Y}\bar{X}_1H_{12} \xrightarrow{\sim} N'/\bar{Y}H_{12} \quad ; \text{ d'où}$$

$$B^N(\Phi_1,\Phi_2) = \int_k \Big[\int_{N'/\bar{Y}H_{12}} \overline{\Phi_1(n'e^{tY})}\ \tau(-at^2/2)\ \ T_{\mathfrak{f}'_2\mathfrak{f}_2}(\Psi_2)(n'e^{tX}2)\ dm($$

Pour établir la majoration cherchée, il suffit de considérer des esp

fonctions denses et on peut donc supposer Φ_1 et $T_{\mathfrak{f}'_2\mathfrak{f}_2}\Psi_2$ décompo

à dire de la forme :

$$\Phi_1(n'e^{tY}) = F'_1(t)\ F''_1(n')$$

$$(T_{\mathfrak{f}'_2\mathfrak{f}_2}\Psi_2)(n'e^{tX}2) = F'_2(t)\ F''_2(n')$$

Lemma 15 *(i) Let A be a maximal σ-split torus and let $\tilde{A}$ be a maxima*
containing A. It is σ-stable (cf. [HH], Lemma 1.9).
(ii) The set of non zero weights of A (resp., $\tilde{A}$) in the Lie algebra of G is a
$\Delta(A)$ (resp., $\Delta(\tilde{A})$) which appears as a subset of $\mathfrak{a}'$(resp., $\tilde{\mathfrak{a}}'$).
The set $\Delta(A)$ is equal to the set of non zero restrictions of elements $\Delta(\tilde{A})$.
(iii) Let $W(A)$ (resp. $W(\tilde{A})$) be the quotient of the normalizer of A (resp.,
(resp. $N_G(\tilde{A})$), by its centralizer, $C_G(A)$ (resp., $C_G(\tilde{A})$).
Then $W(A)$ (resp., $W(\tilde{A})$) identifies with the Weyl group of $\Delta(A)$ (resp.,
is the set of restrictions to $\mathfrak{a}$ of the elements of $W(\tilde{A})$ which normalizes $\mathfrak{a}$.
(iv) Let $\mu,\nu \in \mathfrak{a}'$ which are conjugate by an element of $W(\tilde{A})$, then they ar
by an element of $W(A)$.

Proof :
(i) follows from [HH], Lemma 2.4.
(ii) and (iii) follows from [HW], Propositions 5.3 and 5.9.
(iv) It is clear that one may replace μ and ν by a conjugate element by $W(A)$.

排版手段进入千家万户，这是一个最好的时代，也是最坏的时代.

终极参考书: *The Chicago Manual of Style*, 16th edition (1901–)

常见的排版缺陷

括号高度 试比较

$$(\frac{p}{q})(\frac{q}{p}) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}} \quad \text{与} \quad \left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}.$$

符号忌置句首 考虑句子

"After restriction to X, Λ, μ and Ξ become bounded."

哪个函数限制到哪? 加入赘词可资区分, 如

"After restriction to X, the functions Λ, μ and Ξ become bounded."

慎用下划线 使用下划线是打字机与黑板上的无奈之举, 请善用 **Boldface**, *Italics* 和 Small Caps, 靠字形节奏的变化突出重点.

使用 LaTeX/ConTeXt 排版是一项系统工程. 几项建议:

1. 养成良好的代码习惯: 使用缩排, 注解等.
2. 工具要好使 (WinEdt, Texmaker, TeXstudio, Emacs, vim...).
3. 自动拼字检查.
4. 广识宏包 (上CTAN), 慎选模板.
5. 熟悉绘图及制表的技术, 如 TikZ 宏包.
6. 拒绝急就章.
7. 多方交流, 如 TeX-LaTeX Stack Exchange; `arXiv` 亦可下载源码.

关键: 乐在排版.

本人习惯 (仅供参考): 对于正式论文

1. 原则 — 纸笔先行.
2. 依序制作至少二至三份手稿, 逐步打磨.
3. 过程中检查细节, 并摸索合适的文章结构.
4. 最终稿的标准: 除导言和符号列表外, 须包含所有主干内容; 确保版式整洁, 多年后亦能理解无碍. 【判准之一: 舍不得丢】
5. 根据终稿进行 LaTeX 排版.

此法也适用于整理笔记, 或准备重要讲稿.

Easy reading is damn hard writing.

— *Nathaniel Hawthorne*

补遗

报告的一般要领

确定受众 合理设定背景知识.

克制冲动 由博返约, 删除无关宏旨的细节.

大而化之 保持善意的含混.

由浅入深 “低起点, 高坡度” 的风格适用于多数场合.

掌控节奏 忌仓促, **更忌拖沓**. 统筹设计其中的铺垫, 高潮, 间奏与段子.

割爱的顺序

1. 自己的证明
2. 自己的成果
3. 特别繁琐的细节
4. 众所周知的细节

(注: 暂不适用于国内答辩)

适度使用段子

优秀的讲者 =  + 

无论作诗作词，不可有腐儒气，不可有俗人气，不可有才子气... 尖巧新颖，病在轻薄，发扬暴露，病在浅尽. 腐儒气，俗人气，人犹望而厌之；若才子气，则无不望而悦之矣，故得病最深.

《白雨斋词话 · 卷五》

Beamer 与板书

Beamer/PowerPoint™	板书
信息密度高	信息密度低
便于再利用和传播	很难再利用
单线叙述	多线程, 多“罅隙”
欠生动, 可搭配多媒体	富表演性
极端不适合演示证明	适于演示证明

- Beamer 注意事项
  - 节制公式的使用
  - 坚持技术流
- 板书注意事项
  - 存规矩: 板书 $\neq$ 口语的注解. 善用网格, 避免信马由缰.
  - 求灵活: 板书 $\neq$ 抄书. 多用直觉化的符号.
  - 精进字体修养, 加强辨识度, 如:
    $\xi - \zeta$, $r - \gamma$, $w - \omega$, $6 - \sigma$, $\textcircled{H} - \Theta$...

活用网络: 增加能见度

- arXiv: 流通预印本
- Linkedin, etc.
- 个人主页: 提供履历, 讲义, 投影片, 笑话等...
  - 单位提供
  - 虚拟主机
  - 中国科技网